

B.Sc. (CBCS Pattern) Semester-III
USMT-06 - Mathematics-II : Set Theory and Laplace Transform

P. Pages : 2

Time : Three Hours



GUG/S/25/11613

Max. Marks : 60

- Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that a relation R is symmetric iff $R^{-1} = R$. **6**
- b) Let A be the set of natural numbers n satisfying **6**
 $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$
Show that $A = \mathbb{N}$

OR

- c) Prove that every infinite subset of countable set is countable. **6**
- d) If $a, b \in \mathbb{R}$ then prove that **6**
i) $-(-a) = a$
ii) $-(a+b) = (-a) + (-b)$

UNIT – II

2. a) Find $\tilde{A} \tilde{B}$, where **6**
 $\tilde{A} = \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.6}{4}$ and $\tilde{B} = \frac{0.2}{2} + \frac{0.7}{3} + \frac{0.1}{4}$ are defined on the universal set $U = \{2, 3, 4\}$
- b) Find $\tilde{A} \cap \tilde{B}$, where **6**
 $\tilde{A} = \frac{0.1}{2} + \frac{0.6}{3} + \frac{0.2}{4} + \frac{1}{5} + \frac{0.4}{6} + \frac{0.3}{7} + \frac{0.9}{11}$ and $\tilde{B} = \frac{0.2}{2} + \frac{0.3}{3} + \frac{0.1}{4} + \frac{0.4}{5} + \frac{0.4}{6} + \frac{0.5}{7} + \frac{0.7}{11}$
are defined on the universal set $U = \{2, 3, 4, 5, 6, 7, 11\}$

OR

- c) Let $\tilde{A}_i \in P(U)$ for all $i \in I$, where I is the index set. Then show that $\bigcup_{i \in I}^{\alpha} \tilde{A}_i \subseteq^{\alpha} \left(\bigcup_{i \in I} \tilde{A}_i \right)$ **6**
and $\bigcup_{i \in I}^{\alpha+} \tilde{A}_i =^{\alpha+} \left(\bigcup_{i \in I} \tilde{A}_i \right)$
- d) For any $\tilde{A} \in P(U)$, show that ${}^{\alpha} \tilde{A} = \bigcap_{\beta < \alpha} {}^{\beta} \tilde{A} = \bigcap_{\beta < \alpha} {}^{\beta+} \tilde{A}$ **6**

UNIT – III

3. a) If $L[f(t)] = F(s)$, then prove that $L[e^{at}f(t)] = F(s-a)$. 6
hence find $L[e^{-t} \cosh 2t]$
- b) Find the Laplace transform of $\sin(wt + \alpha)$ and $\cos(wt + \alpha)$ 6
- OR**
- c) Find the Laplace transform of $e^{3t}(\cos 5t - \sin 5t)$ 6
- d) Obtain the Laplace transform of $x''(t) + 2x'(t) - 3x(t)$ where $x(0) = 0, x'(0) = 1$ 6

UNIT – IV

4. a) Find the inverse Laplace transform of $\frac{s^2 - 6}{s^3 + 4s^2 + 3s}$ 6
- b) Find the inverse Laplace transform of $\frac{1}{s(s^2 + 4)}$ by convolution theorem. 6
- OR**
- c) Find the inverse Laplace transform by using convolution theorem 6

$$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$
- d) Solve $(D^4 + D^2 - 2)y = 0$, where $y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1$ 6

5. Solve **any six**

- a) Let $A = \{2, 3, 5\}$, Then find power set of A. 2
- b) Show that $a + b = a + c \Rightarrow b = c \forall a, b, c \in R$ 2
- c) Define support of a fuzzy set. 2
- d) Let $\tilde{A} = \left\{ \frac{0}{a}, \frac{0.4}{b}, \frac{0.5}{c} \right\}$ in $U = \{a, b, c\}$ find $S(\tilde{A})$ 2
- e) Find $L\{|\cos 2t|\}$ 2
- f) Find $L\left\{\int_0^t t^3 dt\right\}$ 2
- g) Find $L^{-1}\left\{\frac{s^2 - 3s + 2}{s^3}\right\}$ 2
- h) Find $L^{-1}\left\{\frac{s}{s^2 a^2 + b^2}\right\}$ 2
